**Problem 1.**

A binary tree T having L leaves must have node of depth at least.

If T is the decision tree for sorting algorithms, then sort n distinct values must be

That means, in the worst case, we need to at least comparisons.

When n = 4: comparisons.

**Problem 2.**

RadixSort of S = {125, 27, 729, 1, 27, 8, 64, 343, 216}, using radix = 9

Observation

Because 729 is max number in S and 729 = 81 \* 9, therefore, we need to change radix from 9 to 10, we have

**k = 100q1 + 10q2 + r**, where 0<= q1 < 10, 0<= q2 < 10, 0<= r < 10

Procedure

With **x** belongs **S** and **x** is inserted first into the slot **x%10** in **r[]** as below

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  | 27 |  |  |
| r[i] |  | 1 |  | 343 | 64 | 125 | 216 | 27 | 8 | 729 |
| i | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

With **y** belongs **r[]** and **y** is inserted first into the slot (**x/10)%10** in **q2[]** as below

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 729 |  |  |  |  |  |  |  |
|  |  |  | 27 |  |  |  |  |  |  |  |
|  | 8 |  | 27 |  |  |  |  |  |  |  |
| q2[j] | 1 | 216 | 125 |  | 343 |  | 64 |  |  |  |
| j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

With **z** belongs **q2[]** and **z** is inserted first into the slot **z/100** in **q1[]** as below

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 64 |  |  |  |  |  |  |  |  |  |
|  | 27 |  |  |  |  |  |  |  |  |  |
|  | 27 |  |  |  |  |  |  |  |  |  |
|  | 8 |  |  |  |  |  |  |  |  |  |
| q1[z] | 1 | 125 | 216 | 343 |  |  |  | 729 |  |  |
| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Sorted order: 1, 8, 27, 27, 64, 125, 216, 343, 729

**Problem 3.**

S = {80, 27, 72, 1, 27, 8, 64, 34, 16}

r[]:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 27 |  |  |  |  |  |  |  |  |
| 72 | 64 |  |  |  |  |  | 16 | 8 |
| 27 | 1 |  |  |  |  |  | 34 | 80 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

q[]:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 34 |  |  |  |  |  |
| 8 |  |  | 27 |  |  |  |  | 80 |
| 0 | 16 |  | 27 |  |  |  | 64 | 72 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Read q[] from left to right, bottom to top: 0, 8, 16, 27, 27, 34, 64, 72, 80